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NUMERICAL INVESTIGATION OF RECIRCULATION FLOWS IN A THREE-DIMENSIONAL CAVERN

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The problem of viscous incompressible fluid flow in a three-dimensional cavity initiated by a moving upper lid is considered. The numerical solution of the Navier–Stokes equations is sought on a grid with diversity velocities in the vector potential-vortex variables. New structures corner vortices and Taylor-Görtler type vortices inherent to three-dimensional flows are obtained numerically. The dependence of the flow nature on the Reynolds number Re and on the ratio between the cavity width to its depth is investigated.

In a number of cases spatial effects can substantially influence the incompressible fluid flow pattern. Consequently solutions obtained when using two-dimensional approximations differ significantly from the experimental data. A typical example is the problem of viscous incompressible fluid flow in a three-dimensional cavity with a moving upper lid. Application of the two-dimensional Navier–Stokes equations assumes that the cavity width L (Fig. 1) is much greater than its depth H. The ratio of the width to the depth of channels varied between 1 and 3 in known experiments [1, 2]. The presence of endface walls and the boundedness of the channel width cause considerable flow reconstruction as compared with the plane case. Numerical computations of viscous fluid flow in a cubic cavern are performed in [3, 4] by using pseudospectral and implicit multigrid methods.

FORMULATION OF FLUID FLOW PROBLEMS IN TWO- AND THREE-DIMENSIONAL CHANNELS WITH A MOVING LID

The problem of two-dimensional fluid flow in a cavity of rectangular section with a moving lid is typical for testing different numerical algorithms [5, 6]. A viscous incompressible fluid flow is examined in a rectangular domain of length B and height H. The fluid is at rest at the initial time, and the upper lid is set in motion at a constant velocity u_0. Adhsesion conditions are given on the cavern boundaries. It is required to determine the stationary laminar flow pattern as a function of Re.

The problem is the following for flows in a three-dimensional cavern. The solution is sought in a domain D (Fig. 1)

$$D = \{(x,\ y,\ z)\colon 0 \leqslant x \leqslant B,\ 0 \leqslant y \leqslant H,\ 0 \leqslant z \leqslant L\}.$$

The moving lid (y = 0) moves from right to left. The boundary conditions are: u(x, 0, z) = 1, v(x, 0, z) = w(x, 0, z) = 0 for y = 0; the velocity vector components u, v, w equal zero on the remaining boundaries. The initial conditions are selected either as at rest (u = v = w = 0) or values of the desired parameters are used for a certain smaller Re.

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TABLE 1

Re	Grid	ψ_{max}	Source
1000	128×128 40×40	0,118 0,112	[6] This paper
3200	128×128 40×40	0,120 0,109	[6] This paper
5000	257×257 60×60	0,119 0,1045	[6] This paper

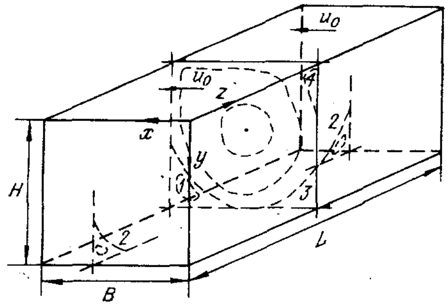

Fig. 1

DESCRIPTION OF THE COMPUTATION ALGORITHM

The equations are written in vector potential-vortex variables in this paper to study the viscous incompressible fluid flows. A numerical algorithm is applied that assures the vortex vector will be solenoidal at each time step and does not require formulation of boundary conditions for it on the solid surface [7, 8].

The question of finding the correct vortical characteristics is of great value in solving incompressible fluid flow problems. Utilization of conservative difference schemes [9] is recommended. It turns out that upon the approximation of the momentum equation (in the two-dimensional case) containing just the conservative terms $\partial\mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V} = 0$, by certain conservative difference schemes (Lax–Wendroff, "with donor cells," MacCormack) keeping the total momentum unchanged in the domain, conservation of the velocity vortex is not assured, fictitious vortex sources and sinks are introduced [10]. Schemes that conserve the vortex during convective transport, are called ω-conservative. It is proposed to use the momentum equation in the Gromeka–Lamb form, for which all the divergent schemes are ω-conservative. The MacCormack scheme is used in this paper for equations of Gromeka–Lamb form.

RESULTS OF TWO-DIMENSIONAL FLOW COMPUTATIONS

The coordinates of the center of the vortex and the intensity of the circulation flow ψ_{max} are important flow characteristics in a two-dimensional channel. A comparison of values found for the intensity of the main circulation flow in a square channel (B = H) with "standard" solutions [6] is presented in Table 1. The velocity profiles are in good agreement. For instance, according to the results of computations in this paper u_{max} = 0.372, 0.412 in the section x = B/2 for Re = 1000 and 5000, while the "standard" values of the maximal velocity are, respectively, 0.383 and 0.436. As the grid becomes finer, convergence of the solution to the "standard" is observed. Thus, the deviation of the found solutions from the "standards" does not exceed 2% in all the parameters on a 80 × 80 cell grid. Utilization of ω-nonconservative schemes results in a noticeable reduction in the circulation flow intensity (by 12% for Re = 1000 and 30% for Re = 5000) and to distortion of the velocity profiles in the sections x = B/2 and y = H/2.

Computations were performed of the stationary flow in a cavity of square section at a given velocity on the upper boundary in the form [5] $u = -16x^2(1 - x)^2$. Values of ψ_{max}, $\max_y u(0.5, y), \int_0^1 \omega(x, 1)dx$ obtained by the authors by using the MacCormack scheme for Re = 400, $\Delta x = \Delta y = 1/20$ are compared in Table 2 with data of computations in [5]. A scheme with more accurate results is represented from a second order scheme [5]. It is seen that the computation data from the proposed algorithm are closer to the results by a fourth-order Hermitian method as compared with others described in [5].

RESULTS OF FLOW COMPUTATIONS IN A THREE-DIMENSIONAL CHANNEL

Detailed data of experimental flow investigations in a three-dimensional cavern with a moving lid are presented in [1, 2]. The main attention in these papers is paid to a study of the flow in the domain of the trailing secondary vortex 1, and specific three-dimensional formations, corner vortices 2 (Fig. 1) and Taylor–Görtler type vortices (TG, Fig. 2).

Numerical computations are performed on a uniform grid with space steps $\Delta = \Delta x = \Delta y = \Delta z = 1/16$. Separate results are obtained on a grid with the step $\Delta = 1/32$. The velocity field in the plane of symmetry is determined by the vector potential component z whose values are used for comparison with two-dimensional computations. Numerical modeling is performed

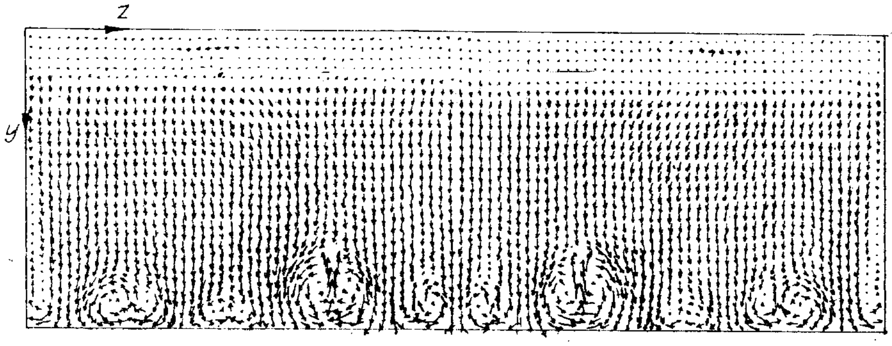

Fig. 2

TABLE 2

Computation method	Ψ_{max}	$\max\limits_{y} u\ (0{,}5,\ y)$	$\int_0^1 \omega\ (x,\ 1)\ dx$
Second-order scheme [5]	0,075	0,188	6,94
Fourth-order Hermitian method [5]	0,0844	0,229	8,10
Method in this paper	0,0784	0,202	7,12

for different Re when B = H, L/H = 1, 2, 3. It showed that the velocity field dynamics in the domain D depends on the ratio L/H and on Re. In the two-dimensional case there is a stationary solution of the problem for Re = 10^4. However, nonstationary modes are realized in the three-dimensional case as Re and the ratio L/H grow.

The Case L/H = 1. For Re = 100 the nature of the flow in the plane of symmetry is the same as for L/H = ∞, however, the intensity of the main circulation zone is 11% less for $\Delta = 1/16$, 12% less for $\Delta = 1/32$ and the trailing 1 and leading 3 secondary vortices (TSV and LSV) have large dimensions and intensity. The transverse flows are weak: on a grid with $\Delta = 1/16 W = \max\limits_{D} w = 0.058$, with $\Delta = 1/32 W = 0.061$, and no corner vortices are detected. For Re = 1000 the intensity of the three-dimensional flow in the plane of symmetry is 39% less than in the plane case for $\Delta = 1/16$, 42% less for $\Delta = 1/32$, which is caused not only by adhesion to the endface walls but also by the growing transverse motions (W = 0.133, 0.151 for $\Delta = 1/16$; 1/32) as well as by the appearance of corner vortices in the sections x = B/2, 3B/4. The TSV and LSV dimensions are less than for Re = 100.

For Re = 2000, a further attenuation occurs in the circulation flow intensity in the plane of symmetry as compared with the variant L/H = ∞, by 46% for $\Delta = 1/16$ and by 52% for $\Delta = 1/32$. The TSV and LSV dimensions in the plane of symmetry diminish as compared with the case Re = 1000, which is in agreement with experimental observations described in [1]. These tendencies hold up to Re = 3300. The solution emerges from the stationary mode in all the modifications.

The Case L/H = 2. For Re = 1000 the flow intensity in the plane of symmetry is 26% less as compared with the two-dimensional modification ($\Delta = 1/16$). Dimensions and the intensity of the corner vortices grow with respect to the modification L/H = 1. The solution emerges into the stationary mode. For Re = 3300, TG type vortices are formed in the sections x = 3B/4 and y = 3H/4, and change location and shape with the lapse of time. For $\Delta = 1/16$ there are from 2 to 5 such vortices at different times, for $\Delta = 1/32$ one or two small vortices of low intensity still appear. The origination of TG type vortices are associated with curvature of the streamlines near the TSV and LSV and with the presence of transverse shear stresses [2]. A study of the velocity fields obtained by using numerical modeling permitted us to establish that such formations appear only in zones of secondary vortex disposition. Strong influence of the TG type vortices on the secondary flow size and intensity is observed. This is felt in the periodic change of the TSV and LSV dimensions in different sections z = const, which is a result of generation and destruction of TG type vortices in parts adjacent to the sections. Changes in the TSV and LSV dimensions are noted in experiments [2]. Attenuation of the flow intensity in the plane of symmetry as compared with the two-dimensional modification is 42% for $\Delta = 1/16$ and 45% for $\Delta = 1/32$. It is caused by efflux of part of the energy into transverse motion being amplified (W = 0.32) and in the formation of the vortical structures described above. The flow mode is nonstationary.

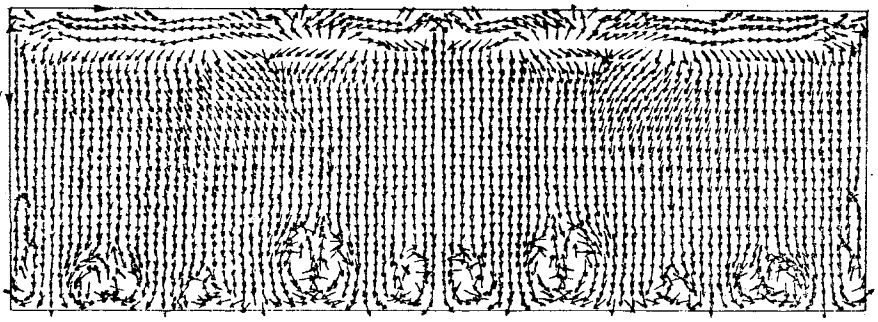

Fig. 3

The Case L/H = 3. For Re = 1000 an intensive fluid flow is formed along the z axis in the plane x = B/2, the dimensions of the corner vortices grow with respect to the case L/H = 2 and the solution is stationary. For $Re \geqslant 2000$, the flow becomes nonstationary, and vortices of TG type occur and vanish in different sections x = const and y = const. The v, w velocity field in the section x = 3B/4 is presented in Fig. 3 for Re = 3300, referred to the maximal value of the velocity in the section mentioned. Only the velocity directions are superposed in Fig. 3.

Vortices of TG type and corner vortices are seen clearly in the figures. A similar flow configuration is observed in experiments [2], where periodic origination and disappearance of the mentioned vortex formations, and the change in time of the TSV and LSV dimensions and intensity are noted. Attenuation of the flow intensity by 30% in the plane of symmetry as compared with the data of two-dimensional computations [6] is also established there. According to results of the authors' computations, the flow intensity in the plane of symmetry is 31% less than in the two-dimensional case on the average.

A detailed analysis of the results of a numerical experiment permitted detection of the presence of TG type vortices in the domain of the upper secondary vortex 4 (see Fig. 1) also for L/H = 3 and Re = 3300.

The computations performed showed that as Re increases for a fixed ratio L/H the flow in a spatial cavity becomes nonstationary, new vortex configurations, vortices of Taylor–Görtler type appear, whose quantity and intensity grow as Re increases. As the ratio of the cavity width to the depth increases from 1 to 3 for a fixed Re, the influence of the endface walls on the circulation flow intensity attenuates in the plane of symmetry while the intensity of the transverse motions increases.

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APPLICATION OF THE REGULARIZATION METHOD TO DETERMINATION OF MULTILAYER STRATA PARAMETERS

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The problem of determining the collector properties of a multilayered petroleum stratum is among the class of inverse problems of underground hydromechanics; it is incorrectly formulated and nonlinear [1, 2]. Questions of the existence and uniqueness of the solution of this problem in the case of radial filtration in the presence of overflows through weakly permeable strata and infiltration were studied in [3]. The problem of determining the collector properties of a monostratum on the basis of the A. N. Tikhonov regularization method was considered in [4]. The present paper is its extension to the case of a multilayer stratum in the presence of overflows through weakly permeable connectors.

1. The majority of petroleum deposits has a laminar configuration due to features of the cumulative settling process. If the ratio of the permeability coefficients of two adjacent seams is less than 10^{-3} then the Myatiev–Girinskii scheme is applicable [1, 2]. We assume known the formulation of the direct problem in formulating the inverse problem. According to the Myatiev–Girinskii scheme the problem to determine the pressure fields $p_1 = p_1(x, y)$ and $p_2 = p_2(x, y)$ in a stratum with nonpermeable roof and floor, separated by a weakly permeable connector reduces under separate exploitation, to solving a system of partial differential equations in a multiconnected domain F with boundaries $\partial D = \Gamma + \sum_{k=1}^{m} \Gamma_k$ (Γ_k are circles of radius $r_s \cong 0.1$ m and centers at the points γ_k)

$$L_1p_1 + \omega(p_1 - p_2) = 0,\ L_1p_1 \equiv -\mathrm{div}(\sigma_1 \mathrm{grad}\, p_1),$$
$$L_2p_2 + \omega(p_2 - p_1) = 0,\ L_2p_2 \equiv -\mathrm{div}(\sigma_2 \mathrm{grad}\, p_2), \qquad (1.1)$$

where σ_i, H_i ($i = 1, 2$) is the hydroconductivity coefficient and thickness of well-permeated seams, $\omega = \sigma_0/H_0^2$, σ_0, H_0 is the hydroconductivity coefficient and thickness of the weakly permeable connector, with the boundary conditions

$$\int_{\Gamma_l} \sigma_k \frac{\partial p_k}{\partial n} ds = q_{kl},\ \left.\frac{\partial p_k}{\partial n}\right|_{\Gamma_l} = 0,\ p_k|_\Gamma = 0,\ k = 1, 2,\ l = 1, 2, \ldots, m, \qquad (1.2)$$

The second of the conditions (1.2) means that the pressure on the contour of each well is constant.

In operator form the boundary value problem (1.1) and (1.2) can be written in the form

$$Lp = 0,\ Mp = Q,\ Np = 0,\ p|_\Gamma = 0.$$

Here $p = (p_1, p_2)$; $L = \begin{pmatrix} L_1 + \omega E & -\omega E \\ -\omega E & L_2 + \omega E \end{pmatrix}$; $M = \{m_{kl}\}$, $N = \{n_{kl}\}$ are 2 × m matrices with elements $m_{kl} = \int_{\Gamma_l} \sigma_k \frac{\partial}{\partial n} ds$, $n_{kl} = \left.\frac{\partial}{\partial n}\right|_{\Gamma_l}$ ($k = 1, 2$, $l = 1, 2, \ldots, m$); $Q = \{q_{kl}\}$ is the matrix of the debits.

The inverse problem is to find the quantities σ_0, σ_1, σ_2. Its initial data are the given debits $q_{k\ell}$, the values of the face pressure $p^*_{kl} = p_k|_{\Gamma_l}$ ($k = 1, 2$, $l = 1, 2, \ldots, m$, $m \geqslant 2$) and the pressure functions on the boundary of the filtration domain. This inverse problem generates a certain implicitly given nonlinear operator

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